SIMULATION WITH SYSTEM DYNAMICS AND FUZZY REASONING OF A TAX POLICY TO REDUCE CO₂ EMISSIONS IN THE RESIDENTIAL SECTOR

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Abstract

The methodological paper presents a strategic model to simulate the reduction of CO₂ emissions caused by the heating and lighting in the residential sector. The objective is to illustrate with this practical case a system-dynamics based simulation methodology in uncertain future, developed for providing policy recommendations to public decision-makers. The uncertainties on the marginal cost curves and the macroeconomic impacts of the tax policy are modelled by fuzzy rules established by experts with defined credibility levels.

Keywords: system dynamics, fuzzy reasoning, environmental management, CO₂-emission reduction

1. Introduction – Objective of the paper

This paper presents a planning and control methodology for assisting public policy-makers in defining adequate tax levels for reducing CO₂-emissions caused by heating and lighting in the residential home sector. We do not enter here into a specific discussion regarding a particular territory or a given specific case. Our purpose is entirely a methodological one, though we present a simplified didactic example illustrating the proposed approach.

The presented methodology consists in including fuzzy-reasoning techniques to the ‘Adaptive Control Methodology’ (ACM) previously developed in a purely deterministic concept in Brans et al. (1998), Kunsch et al. (1999, 2001). The ACM is a decision-aiding technique combining system dynamics with multicriteria decision aid to assist policy-makers to make decisions in complex socio-economic systems. In this paper we do not explicitly consider the multicriteria dimension of the original ACM, as our main purpose is to describe the treatment of uncertainties which is proposed in the SD model.

The scheme, which is developed to change the behaviour of residential consumers, consists in applying a unit tax to each weight unit of fossil fuel to be burnt for producing electricity or heat in homes.

Two basic questions are to be answered by the authorities:

- Which level of fuel-consumption reduction is realistic in the home sector? With other words, which objective in percentage reduction of the use of fossil fuel will be achievable and economically sound?

- Which dynamic unit tax should be added to the price of fuel in order to be efficient in approaching the proposed reduction objective, while avoiding at the same time too large price increases and unwanted macroeconomic impacts?

Answering the first question requires a sufficient knowledge on the marginal costs of fuel reduction, to be able to evaluate the willingness to invest in less fuel-consuming technologies. Answering the second question requires some conviction regarding feedback loops between increasing energy prices and interest rates. Both aspects are imprecisely known and must rely on judgements of experts which are often diverging and are more or less credible.
But the incapacity in stating clear values of marginal costs or well-cut relationships between macroeconomic variables is pervasive in complex human systems. Because statistical data are scarce or inexistente, it is often difficult, if not impossible, to base sound conclusions on statistical analyses. It is why we propose to base decision-making on expert opinions. We argue that fuzzy reasoning is an adequate tool for aggregating in a convincing way diverging expert judgements on these matters.

The paper is constructed as follows.

In section 2 we give the main principles useful for understanding tax schemes on fossil fuel consumption to achieve CO\textsubscript{2}-reductions. Section 3 describes the expected effect of the tax on the fuel consumption in the residential sector. Section 4 develops a simplified didactic model elaborated on these premises. The nature of uncertainties on marginal cost curves and impacts of the fuel price with tax on the macroeconomic interest rate are discussed. Section 5 gives a general overview on fuzzy reasoning to model decision-making in uncertain futures. The proposed approach is then explained in relation to the different expert opinions on uncertain parameters in the simplified model. Conclusions are given in section 6.

2. Principles of the fuel tax

There are different instruments developed to control pollution, in particular CO\textsubscript{2} emissions:

1) Command-and-Control;
2) Tax or subsidies;
3) Licences or marketable emission permits.

The Command-and-Control regime consists in imposing limits on emissions by means of regulations imposed by a state-owned regulating body. We will not further discuss here this possibility.

A tax regime is another instrument handed by the state. A tax per unit of emitted pollutant of a given kind, e.g. CO\textsubscript{2} or CO\textsubscript{2}-equivalent, is charged to the polluters. The principle is shown in figure 1. It is assumed that a curve (here represented by a straight line for simplicity) is perfectly known representing the marginal abatement cost (MC) per unit of emission, in function of the total emission level of the specific pollutant (M).

![Figure 1. A handbook representation of the marginal abatement cost (MC) in function of the pollution level (M). T* represents the tax level; the optimal emission level for this value of the tax is given at M_T. The zero-abatement level is given at M_0.](image)

In the classical handbook representation like Hanley at al. (1997), Perman et al. (2003), this curve would be decreasing to the maximum emission level per time unit (M_0), because of the
law of diminishing return. The tax rate 
M_0 corresponds to the situation where no abatement measure is taken. A rational polluter would decrease the emission level from M_0 to M_T in order to achieve the economic optimum (see Perman et al. (2003) for a formal proof). The polluter has to pay the tax amount indicated by the rectangular surface. At equilibrium the total emission will be calculated as follows, given a tax level T*, and inverting the function MC(M):

\[ M_T = MC^{-1}(T*) \] (1)

A subsidy regime, not further discussed here, would be a mirror image of the tax regime: it encourages abatement measures by a positive payment from the state regulator to the polluter, up to an equilibrium point between marginal cost and per-unit subsidy.

A scheme of marketable emission permits is a mixed scheme, combining state interventions and the mechanisms of the market. It functions on the basis of the ‘cap-and-trade’ principle (see IEA (2001)). This means that each year the regulatory authority caps the yearly emission level which is allowed by issuing the corresponding number of permits. The idea is to have year after year a decline in this emission level. Permits can be traded on a specialised permit market between the users, who are the potential polluters. Each permit has a nominal value, expressed in units of pollutant emission per year, e.g., 1 metric ton CO\textsubscript{2}/year. It herewith gives the right to his possessor to emit this quantity during the year. This implies that a given polluting operator receives the permission to emit an annual total quantity given by the total number of permit he has in hands, times the facial value of each permits, in general metric tons of pollutants per year.

Tax and emission-permit schemes have much in common, because their functioning depend on the good knowledge of the MC curve (see Kunsch et al. (2004)). We will not discuss the permit scheme here any further. The reader is referred to Kunsch and Springael (2004), the companion paper to this one. It is devoted to calculating the evolution of the market price of permits, using a similar fuzzy-reasoning methodology. We have used the same technique as in this paper to define aggregated MC curves.

Both schemes have advantages and drawbacks, as also shown in the companion paper. An important drawback of taxes is that it is difficult to choose with any precision the most efficient tax level for achieving some predefined objective M_T (see figure 1). This is because a tax scheme operates on a cost basis, without precise consideration of the abatement goal. Assume in figure 1 that the MC curve is displaced horizontally to the right because of uncertainties. The equilibrium emission level M_T will be translated to the right in the same way, giving a different policy result.

Permits are in this respect more reliable and flexible, while the uncertainty on the MC curve will not affect the objective but only the final permit price on the market.

Note that in the case of CO\textsubscript{2} emission in the residential sector, the practical way is to charge a so-called energy tax on the fossil fuel emitting CO\textsubscript{2}, rather than on the emissions themselves. The effects of the tax must thus be analysed on the basis of the consumer behaviour in reducing his consumption of fossil fuel. The presentation of the MC curve of figure 1 must be adapted to this situation:

- The horizontal axis ‘r’ now represents the reduction in fossil fuel consumption. The origin at r=0 represents the initial conditions M_0 in figure 1 for which no reduction has yet taken place;

- For reasons of convenience, we assume a yearly reduction in quantity of fossil fuel, expressed in the units [w\textsubscript{fuel}/year] (where ‘w’ says for weight) equivalent to a unit reduction in CO\textsubscript{2} emissions, expressed in [tCO\textsubscript{2}/year]
We thus have the following equivalence:

\[ 1 \text{ wfuel/year less consumed is equivalent to } 1 \text{tCO}_2/\text{year less emitted.} \]  

(2)

3. Modelling a tax policy in the residential sector

In this section we assume that consumers are rational investors following the rules of Cost-Benefit analysis (CBA). Of course this assumption could be criticised, but it allows us to introduce our approach in a simple way. More refined assumptions are easily implemented in the simple investment model we now describe.

Call ‘i’ the interest rate prevailing in the economy for borrowing money with the intention to invest into less polluting equipment in households. Elementary CBA teaches us that the cost of initial investment must be smaller or equal to the stream of discounted future benefits (Mishan (1988)). Thus:

\[ \text{Cost of reducing one unit of fuel} = \text{Benefits of reducing one unit of fuel} \]  

(3)

1) The cost of reducing one unit of (fossil) fuel is equal by definition to the marginal cost of reduction MC, expressed in currency units/year [CU/year], so that:

\[ \text{Anticipated cost of fuel reduction} (t) = MC(t) \]  

(4)

Note here that the policy-makers scrutinising the abatement potential are not directly interested in the many individuals’ MC curves. Rather, quite few MC curves of interest can be identified. They are common to similar techniques or technologies used by residential homeowners for reducing their fossil-fuel consumptions.

2) The benefit of a unit reduction per year corresponds to an infinite stream of cost reductions attached to one reduced weight unit of fuel per year, the value of which is given by the well-know formula of perpetuity:

\[ \text{Anticipated stream of benefits of fuel unit reduction} \text{ per year} (t) = \frac{P_{w/o}(t)}{i(t)} \]  

(5)

Where \( P_{w/o} \) [CU/wfuel] is the unit price of fuel at time \( t \) including the tax; and \( i(t) \) [%/year] is the yearly interest rate in the economy used for borrowing money.

By comparing equations (4) and (5) we obtain the condition for the consumers to be willing to invest in fossil-fuel reduction or substitution measures in their homes:

\[ MC(t) \leq (P_{w/o} + ut)(t) / i(t) \]  

(6)

where \( P_{w/o} \) [CU/wfuel] is the unit price of fuel on the market at time \( t \) excluding the unit tax \( ut \) [CU/wfuel]. It is an exogenous variable to the model.

Equation (6) is the basis of our model. MC curves when plotted in function of the reduction of the equivalent weight of fuel per year [wfuel/year] may have complex shapes, but sooner or later they are surging up to vertical. The economically achievable fossil-fuel reduction will be located in the bow to vertical. Figure 2 shows that the MC curve which will be used in the didactic example later in the paper have these characteristics. It will be explained how it is obtained by aggregating different reduction measures and several expert opinions.
Looking at this curve, it can be observed that a reasonable limit \( L \) for the yearly fuel reduction would be around \( L = 50\text{ wfuel/year} \). This gives to policy-makers the information necessary to design a tax policy, assuming that they plan a time-horizon of \( T \) years to realise this reduction objective. Combining these two parameters thus provides a guideline \( G(t) \) in [wfuel/year] for the dynamic reduction of fossil fuel \( r(t) \) over \( T \) years.

Assuming a linear evolution of the guideline with rate \( g(t) \) we obtain a constant value:

\[
g(t) = \frac{dG(t)}{dt} = \frac{L}{T} \frac{\text{wfuel}}{\text{year}^2}
\]  

(7)

This guideline will not be exactly respected because there are some delays in realising it, as will be shown later. It can be approached by adjusting the price at time \( t \) to its theoretical control value to match \( G(t) \).

This theoretical control price will be given according to equation (6) by the following equality:

\[
P_{th} = i.MC \left[ G(t) \right]
\]  

(8)

The laws of economy define the fossil-fuel price without tax \( P_{w/o} \); in general, it will be fluctuating around a medium-term trend. The unit tax \( ut(t) \) can be adjusted using proportional control, common in system-dynamics models, using the adjustment time \( T_{adj} \) as follows:
\[
\frac{du(t)}{dt} = (P_w - P_{w/0})(t)
\]
\[
\frac{dP_w(t)}{dt} = \frac{(P_m - P_w)(t)}{T_{adj}}
\]  

(9)

Remember that this equation (9) corresponds in SD to an exponential information delay of the first order (see Sterman (2000), chapter 11).

Combining equation (6) and (8) we can calculate the evolution of fossil-fuel usage. For that purpose, we first introduce the variable ‘Willingness To Invest” (WTI) which expresses that the decision to invest passes the CBA test of equation (6):

WTI is equal to:

\[
\begin{align*}
\text{WTI} &= 0 & \text{if } P_w &< i.MC.(1 - \varepsilon) \\
\text{WTI} &= 1 & \text{if } P_w &\geq i.MC.(1 - \varepsilon)
\end{align*}
\]  

(10)

where \(\varepsilon\) represents a tolerance for the CBA test, e.g. 0.5% is a reasonable value.

When WTI=1, investment is economically founded and will thus take place according to our rationality hypothesis.

In our model we are not able to invert the MC curve (see equation (1)). To determine the reduction value \(r(t)\) that will result from the use of a fuel price \(P_w(t)\), we use a calculation trick:

The evolution of \(r(t)\) will be along the MC curve, so that, when WTI=1:

\[
\frac{dMC[r(t)]}{dt} = \frac{dMC}{dr} \frac{dr}{dt}
\]

\[
\rightarrow \frac{dr}{dt} = \frac{dMC/}{dt} \frac{dMC/}{dr}
\]  

(11)

When WTI=1, the evolution of \(\frac{dMC}{dt}\) is dictated by the price, with the evolution given by equation (9). A distinction must however be made between the two possible cases: either \(P_w\) is equal to (within the tolerance \(\varepsilon\)), or larger than \(i.MC\):

While WTI =1

EITHER \(P_w = i.MC\)

\[
\frac{dr}{dt} = \frac{dMC/}{dt} = \frac{P_w/}{dt} \frac{dMC/}{dr} = \frac{P_{th} - P_w}{T_{adj} i. dMC/} \frac{dMC/}{dr}
\]  

(12)
Thus in case \( P_w \) is larger than \( i.MC \), the fuel reduction is dictated by the rate of change of the guideline in equation (7): the MC will move up to the price/interest rate at this rate of change.

Two simplifications appear in this model:

1) The increase of the total price with tax may be capped by some price-control authority. Let us call \( P_{max} \) this maximum admissible value. This constraint is represented by adding to equation (9) a Verhulst-type factor (see Sterman (2000), p. 296):

\[
\frac{dP_w}{dt} = \frac{(P_{th} - P_w)}{T_{adj}} (1 - \frac{P_w}{P_{max}})
\]

The value of \( P_{max} \) is subject to a decision of the authorities, which may require some form of political consensus. A simple way of determining this price is to read from the MC curve on figure 3 the MC value corresponding to the reduction limit \( L \) in equation (7), and to multiply it with the initial interest rate. In this particular case we would obtain \( P_{max}=600 \) CUI/(w fuel/year), corresponding to the maximum economically feasible reduction \( L=50 \) w fuel/year, and \( i_0=4\% \), assumed in our example.

2) The increase of the fuel price to achieve the reduction guideline is not without macroeconomic consequences. The more expensive the fuel, the less economic growth is to be expected; the more expensive it will be for private investors to borrow money. A direct link can be expected between the fuel price and the interest rate. This creates an important positive feedback loop driving up the price still further. Of course it is not a simple task to formalise in the model the exact dependence between both variables. Many opinions of expert economists do exist in this respect, creating another source of uncertainty.

In our model we have included the following relationship for the sake of describing our approach (we do not claim that real experts would not come with different rules):

\[
\Delta I_P = f \left( I = \frac{P_u - P_0}{P_{max} - P_0} \right) > 0
\]

\[0 \leq f(.) \leq 1 \text{ ; } 0 \leq I \leq 1\]

\( \Delta I_P \) is the change in the current interest rate due the change in fuel price with tax \( P_w \). \( P_0 \) is the initial fuel price with tax. I, calculated, as shown in the argument of the function \( f(.) \), is called the ‘impact factor’.

The ‘impact function’ \( f(.) \) is non-linear and increasing with the impact factor \( I \).

The total interest rate can then be represented as a random walk process, for example (see Hull(2000) pp. 220-226):

\[
i(t) = i(t_0) + \mu (t-t_0) + \sigma dz + \Delta I_P
\]
where $\mu$ represents the trend in the interest rate changing over time, $\sigma \, dz$ is a Wiener process of standard deviation $\sigma$, $\Delta t_p$ is defined inside the model in equation (15). Our model must be able to adequately respond to a random exogenous evolution like represented in the three first terms on the RHS of equation (16).

4. A simple didactic model

4.1 The deterministic model

The deterministic model of Kunsch et al. (1999, 2001) was extremely simplified with respect to the tax model and it took no uncertainties into account. It has thus been developed to include the full tax model presented in section 3, and to give allowance to the uncertainty model to be presented in the following section.

The influence diagram of the System-Dynamics (SD) model is reproduced in figure 3 (see Sterman (2000) for details on SD-modelling). It gives an overview of the deterministic tax model. Two main feedback loops are represented in bold lines.

- The reduction objective, or rather the guideline introduced in equation (7), is calculated in the upper part of the diagram. It functions as a feed forward in the model, triggering the adjustment in the fuel price by means of the tax. We have adopted in this simple example a maximum reduction $L=50 \, \text{w fuel/year}$, as explained earlier in the text;
- Several stocks and negative control loops are found in the central part of the diagram model: the price adjustment to the theoretical price described in equation (8) and (9); the ‘Willingness To Invest’ according to the CBA test (10); finally the evolution of the fossil-fuel reduction along the MC curve according to equation (11) and (12);
- The macroeconomic impact of the fuel-price increase according to equations (14) and (15) is visible in the lower part of the diagram. It manifests itself by the existence of a positive destabilising loop on the interest rate (its initial value is chosen to be 4%/year);
- The calculation of the CO$_2$-emissions is in the upper right part of the diagram.

To determine the MC curve (figure 2) in this calculation, it has been supposed that three techniques are used in the residential sector to reduce the fossil-fuel consumption of private residents:

- RUE (Rational Use of energy), for example the use of long-lived and less consuming lighting bulbs, automatic light switches, etc.;
- HED (high efficiency devices), for example double-panel windows, efficient furnaces or refrigerators, etc.;
- GE (Green electricity) use of renewable electricity (biomass, wind energy) for heating and lighting.

The resulting MC curve introduced in figure 2 has been computed by means on the methodology based on expert opinions aggregated by fuzzy reasoning we will introduce in the next section. In establishing this curve a complication in the approach arises because of the existence of the three technologies RUE, HED, GE, and the need to collapse three MC curves into one. The elaboration of all necessary steps in this case is beyond the scope of this paper. The reader is referred to the original paper for details (Kunsch and Springael, op. cit.). We thus assume from now on that the global MC curve has been made available in a prior elaboration step.

We assume that two experts (expert 1 and expert 2) provide each the impact function defined in equation (15). Both functions are represented in figure 4. The maximum increase in the interest rate up to 2.5% is assumed by expert 1. The second expert assumes a far weaker
influence of the impact factor $I$ on the interest rate, manifesting herewith that the actual behaviour is largely unknown. In this first deterministic calculation only the opinion of expert 1 is retained.

![Graph Lookup - interest rate expert 1 lookup](image1)

Figure 3. The system-dynamics model in the deterministic case with a unique expert.

![Graph Lookup - interest rate expert 2 lookup](image2)

Figure 4. The rather different impact functions predicted by two experts $f_1(I)$, and $f_2(I)$ linking, as shown in equation (15), the impact factor $I$ lying in the interval $[0, 1]$ to the increase in the interest rate $\Delta i_p$, with a maximum of 2.5% for the first expert.

Figure 5 shows some results of the computation when only expert 1 is asked for an opinion, i.e. when there are no uncertainties on the impact function are considered.

The upper left time graph shows the fossil-fuel reduction path lagging behind the guideline $G(t)$ defined in equation (7). This is due to the delays present in the system, e.g. in equation (9) corresponding to a first-order delay.
The upper right time graph shows how the fuel-price with included tax will have to increase, in order to follow as closely as possible the guideline according to equation (9). This price, increase, expressed in [CU/wfuel], favours the investment willingness of homeowners, as shown in the bottom left figure representing the investment rate in [CU/year]. It is a rather erratic behaviour depending on the CBA test on the ‘Willingness to Invest’ (WTI) variable in equation (10). According to equation (15) another effect of the price increase is to bring the interest rate from its initial 4%/year value to higher values close to 6.5%/year, as shown in the bottom right graph. The effect of the positive feedback loop evidenced in figure 3 brings in turn the price further up.

Figure 5. Some results of the deterministic simulations with the SD-model of figure 3 for a unique expert.

4.2 Modelling uncertainties on the impact function

Assume now that the opinion of expert2 is asked regarding the impact function (see figure 4). Though the eventual reduction path comes out as nearly the same as for expert1’s opinion, the price policy, and thus the tax evolution will be quite different, as shown in figure 6. These results prove the overwhelming importance, for designing an efficient tax scheme, of having a sufficient knowledge of basic variables (like the MC curves see the paper by Kunsch and Springael (op. cit.), and/or of functional relationships between key variables within the dynamic model. In practice this knowledge rests on expert opinions, with varying credibility degrees, and often-diverging results. As shown in figure 6, a wrong assessment will in fact give unpredictable results for the tax schemes. For example, using the lower tax curve in the right part of this figure will give very poor results indeed, if in reality the actual evolution compatible with the guideline rather fits the upper tax curve.

Tools must be developed to aggregate diverging opinions of this kind, taking into account their respective degrees of credibility. In the next section we present the approach we have developed for better achieving this objective.
Figure 6. The influence of the opinions of two experts on the impact functions (see figure 4). The upper curves represent the resulting interest rate (on the left) and unit tax on fuel (on the right) resulting from the opinion of the first experts. The results according to the second expert are accordingly in the lower curves.

In order to be able to model uncertainties of the type described, e.g. the MC curves or the impact function, it is necessary to gather the broadest possible range of expert opinions on each important and uncertain aspect. These opinions are the equivalent of a range of scenarios, but there is no need for a-priori subjective probabilities, like in the Bayesian approach. It is only assumed that a scoring of the experts on a [0, 1] scale is available. It should be derived from their experience in energy conservation, efficient use, and past performances of environment-friendly devices, etc. It is beyond the scope of this paper to discuss how the scoring of experts is made. The readers are referred to the existing literature on this aspect like described in Meyer and Booker (2001). The scoring expresses the credibility that can be given to opinions. It can be decided to eliminate less useful opinions, i.e., scenarios, by using cut-off rules, for example eliminating all opinions that score less than 0.25, etc. The same approach can of course be used by giving directly credibility scores to scenarios for available historical data, but this seems to be a more perilous attempt. The Past is only a poor predictor of the Future. It is thus better to use multiple human opinions as basic inputs.

Scores of experts are NOT simply used as weights. A more sophisticated and less arbitrary aggregation techniques than weighed sum must be developed. It is again based on fuzzy reasoning we now introduce.

5. A fuzzy–reasoning approach to aggregate expert opinions

In Kunsch and Fortemps (2002), one of the authors has discussed approaches in fuzzy reasoning to aggregate expert opinions. We use some explanatory material given in this paper to introduce this technique.

Fuzzy Logic (FL) is a mathematical technique to assist decisions on the basis of rather vague statements and logical implications between variables. FL is close to the natural language, this is why some people have called it ‘computation with words’. FL is very useful in many technical and economic applications in which imprecise and relatively vague judgements of experts have to be accounted for in a quantitative way as explained for business applications in Cox (1995).

The first step in the approach is called ‘Fuzzification’.

The basic ingredients of fuzzification are (1) ‘membership functions’ to represent range of possible values of a vague or imprecisely known variable (‘fuzzy variable’ as opposed to ‘crisp variable’), and (2) ‘fuzzy rules’. The latter relate fuzzy variables, in the antecedent of
the rule on its input side, to draw some conclusions on the final results, in the consequent of the rule.

(1) A ‘Membership function’ (m.f.) provides a possibility measure, called ‘membership grade’ (m.g.) for some affirmation. For example, the m.f. ‘MIDDLEAGED’ for a human being might be represented by a triangular m.f. as follows: the m.g. is 0 at 30 years (y), it peaks at 1 at 45 y, and it comes down to 0 at 60 y. This triangular m.f. is represented by the triple (30 y; 45 y; 60 y). In the same context other lifetimes could be represented, e.g., ‘CHILD’, ‘YOUNG’, ‘OLD’. The interval of variation of the fuzzy variable is called the universe of discourse, in the given example for life-ages, it could be in the interval [0, 100] (years). The first part of fuzzification consists in translating imprecise variables into a fuzzy variable, represented by a m.f., e.g. using four m.f.’s describing different ages of life. Note that m.f.’s are different from probability distributions. For example, the total surface underneath any m.f. is not normalised to 1. They are defined in the framework of possibility distributions.

(2) A mapping between fuzzy variables is made possible by using ‘fuzzy rules’, which are the second part of fuzzification. In the given example, rules connecting the life-ages to the degrees of experience could be imagined:

(a) If ‘AGE’ is ‘YOUNG’ then ‘EXPERIENCE’ is ‘LIMITED’
(b) If ‘AGE’ is ‘MIDDLE-AGED’ then ‘EXPERIENCE’ is ‘APPRECIABLE’

etc.

In this 1-input, 1-output fuzzy system, the 4 life-ages (‘CHILD’, ‘YOUNG’, MIDDLEAGED’, and ‘OLD’) may be represented by triangular m.f.’s and the experience levels by corresponding four trapezoidal m.f.’s (e.g., ‘VANISHING’, ‘LIMITED’, ‘APPRECIABLE’, ‘IMPORTANT’).

The second step is called ‘Implication’.

An implication operator defines the m.f.’s of consequents, given some value of the antecedent and applying the logical fuzzy rules. The ‘min’ implication operator, corresponding to a logical ‘AND’ is commonly used in control systems of the Mandami or Sugeno type as explained in Passino and Yurkovich (1998).

For example, assuming that u is the m.g. of the input ‘YOUNG’ to the rule (a), and Y is the m.f. representing ‘LIMITED’ the ‘min’ implication operator will give as output for the rule a truncated trapezoidal m.f. of height min(u,Y).

The third step in fuzzy reasoning is called ‘Aggregation’.

In aggregation, the consequents of all partial rules are aggregated using an additional aggregation operator, e.g. the ‘max’ operator corresponding to a logical ‘OR’. This operation will result in a composite m.f..

The fourth and final step is called ‘Defuzzification’.

Defuzzification consists in deriving a unique final answer from the composite m.f. obtained in the aggregation. Different defuzzification operators are used. The most common one is ‘centroid’ which comes to calculating the center-of-gravity of the aggregated m.f..
The same sequence of four fuzzy-reasoning steps, ‘fuzzification’, ‘implication’, ‘aggregation’, ‘defuzzification’ will be used in the particular reduction problem of fossil-fuel usage we have presented here. As said before in Kunsch and Springael (op. cit.) the authors elaborated the calculation of the MC curve, like shown in figure 2. It is based on the three technologies, RUE, HED, and GE on which two well-contrasted experts are assumed to express opinions (of course any number of technologies and experts can be assumed).

To avoid repetitions we concentrate on the elaboration of the uncertain impact function defined in equation (15).

Assume that there are n experts, and that their credibility factors \( C_i \) are given on a [0, 1] scale.

To follow the described sequence in fuzzy reasoning we start with ‘Fuzzification’. The universe of discourse for representing the impact factor \( I \) (see equation (15) is of course [0, 1], independently of the opinions of experts (assuming that all of them agree on the choice of \( P_{\text{max}} \) in this equation). We can thus define \( L \) levels, covering the [0, 1] interval, each represented by a triangular m.f. \( u_l \), \( l = 1, L \). For reasons of convenience, we take \( L = 5 \):

\[
( u_l ; l = 1, 5 ; \text{vanishing, small, medium, large, absolute})
\]

(17)

These five m.f.’s appear on the left of figure 7 we will discuss in detail.

Each expert makes a mapping of these levels to the levels of the impact function \( f(. \), representing the change of the interest rate (IR) from its initial value. Each mapping generates a set of \( L = 5 \) membership functions, each representing an opinion \( O \) on the IR, expressed in [%/year]. This provides in all \( n \times L = 5 \) expert opinions, as follows:

\[
O(i=1,n; l=1,L; \text{Opinion of n experts on the IR}(l) \text{ for } l=1, L \text{ l-levels}) = O(i,l)
\]

(18)

They correspond to the set of \( n \times L \) partial rules:

\[
\text{IF Impact Factor is } I(l) \text{. AND. Expert}(i) \text{ is } C_i \text{, THEN IR}(l) \text{ is } O(i,l)
\]

(19)

(In this particular case the opinions correspond to the impact functions in figure 4.)

Equations (17) to (19) complete ‘fuzzification’.

What we are now up to is to perform the second step, i.e. ‘Implication’. It consists in calculating the m.f. of the conclusion of each partial rule.

To make things simple we use here the so-called Mamdani-Sugeno implication explained in Passino and Yurkovich (op. cit.). This implication, say \( R_{MS} \), is the conjunction with the logical ‘AND’, represented by the simple ‘min’ operator between the inputs and the output of the rule. Calling \( \mu_i \) the m.f. of the conclusion of the partial rule \((i,l)\) (equation (19)), established by expert \( i \) for the \( l \)-th level of the impact factor \( I \), we may write:

\[
\mu_i = R_{MS}(u_i , \nu_i) = \min(u_i , \nu_i)
\]

(20)

where
\( u_{il} \) represents the membership grade (m.g.) of the antecedent to the rule \((i,l)\) in equation (19);
\( v_{il} \) represents the membership function (m.f.) of the opinion \(O(i,l)\) coming in the conclusion of the rule in equation (19).

The antecedent of the rule \((i,l)\) in equation (19) is itself the conjunction of two inputs. The first one on the left, represents the m.g. of the \(l\)-th I-level (for example for \(I(l=3)\), \(u_3=0.4\)), the second one on the right represents the credibility \(C_i\) of the \(i\)-th expert (for example \(C_2=0.7\)). Because this is a conjunction, the simple min-operator applied to these two values can be used: both values are by definition in the interval \([0,1]\).

In this case we obtain for \(u_{il}\) defined in equation (20)

\[
u_{il} = \min(u_i, C_i)
\]

(In this example \(u_{23} = \min(0.4; 0.7) = 0.4\), for the I-level \(l=3\) (‘medium’) in the judgement of the expert \(i=2\)).

Note from (21) that an expert with a vanishing credibility will have a vanishing m.f. for all opinions he expresses, by application of the implication (20). This expert will thus be ignored in the further treatment. If all experts have a vanishing credibility, no conclusion can be drawn at all from fuzzy reasoning.

For the m.f. \(v_{il}\), defined in (20), it is sufficient to adopt as a single value of the marginal cost (singleton with m.g.=1). This is the special form of the Mamdani-Sugeno inference introduced by Sugeno, and thus called more simply Sugeno implication (Passino and Yurkovich (op. cit.)). It is very useful to represent arbitrary functions in control theory, like MC-curves, or the impact function, the shape of which can be complicated. The singleton receives the m.g. \(v_{il} = 1\), so that equation (20) immediately simplifies to:

\[
u_{il} = u_{il}
\]

In the Sugeno implication the conclusion of each rule thus receives the same m.g. as the antecedent of the rule.

Figure 7. The fuzzy-reasoning steps on the impact function defined in equation (15) for \(n=2\) experts \((C_1=0.5; C_2=0.7)\) are from left to right: ‘Fuzzification’ of three inputs \((I, C_1, C_2)\); ‘Implication’ of the partial Sugeno-rules, ‘Aggregation’ of the partial conclusions of rules to a global m.f. (right-bottom window), and ‘Defuzzification’ of the global m.f. to a unique output through the center-of-gravity methodology (output1=0.67%) (arbitrary scales and units). (Calculations are made with Fuzzy Toolbox of MATLAB® (2001)).
This process is well visible on the right of figure 7 representing the full fuzzy-reasoning process. Each window represents a rule. For simplification, two experts are considered (n=2). Because of L=5, there are 10 rules, each represented in a separate window. The membership grade of the combined inputs is calculated by means of the conjunction operator ‘min’ on the left. The membership functions of the conclusions on the left reduce to singletons which receive the same membership grade as the combined inputs of the applicable rule. These singletons are obtained for each expert by considering the five ‘anchor values’ (I(l); l=1..5) of the impact functions of figure 4.

The next step is ‘Aggregation’. The conclusions of all partial rules (i, l) relative to all experts and all impact-function levels are combined in order to obtain a global m.f.. This is done by using the logical ‘OR’, represented by the simple ‘max’ operator applied to all outputs of the individual rules. In the Sugeno inference this is particularly simple, because the conclusion of each rule is a singleton: the m.g. has just been calculated in the previous implication step. The global aggregated m.f. has thus n*L components given by:

\[ m.g. (I) = \{ u_{il} ; i = 1, n ; l = 1, L \} \]

The lowest frame on the right of figure 7 represents the set of values constituting the global m.f.. The latter aggregates all partial rules.

The final step in the fuzzy-reasoning schemes is ‘Defuzzification’ of the global m.f.’s. It consists in calculating a unique ‘crisp’ value from the global m.f. for the interest rate increase \( \Delta i_p \). In this particular case the center-of-gravity (COG) is the most adapted approach to obtain this value. This is expressed as follows for each set of inputs (impact factor and credibility factors):

\[ COG \left[ I(l), l = 1, L; C_i, i = 1, n \right] = \frac{\sum_{i=1,n}^{L} O(i,l) u_{il}}{\sum_{i=1,n; l=1,L} u_{il}} \]

The calculation of the COG is shown in the lowermost window on the left of figure 7, giving a current value \( \Delta i_p = 0.67\% \) for the increase of the interest rate.

Note that fuzzy-inference systems (FIS) like just described have the property of being universal approximators for any non-linear function. In the case of only one expert (n=1), equation (24) is shown to interpolate the impact function between the anchor values, i.e., the opinions this expert has given for the L impact-factor I-levels. For a very complex impact function the number of anchor points and thus the number of levels L and of rules may have to be kept quite large, in order to stick as closely as possible to the expected function. In this case n=1, which corresponds to the absence of uncertainties on the MC curves, limited information is brought by the use of fuzzy reasoning. The most direct way is to use the suitably interpolated MC curve.

The added value of fuzzy reasoning comes with the existence of different opinions. Formula (24) then do not only interpolate between anchor points. It provides in addition an easy approach for aggregating all opinions to one global result, taking into account the credibility grade of each opinion. We remark that regarding the credibility grades of experts, this approach is far less arbitrary than a simple additive weighing technique for combining opinions! The sum of credibility factors is not normalised. If necessary, additional stochastic risk analysis can be added to this simulation model, e.g. as follows:
- Budgets can be attached to the guideline $G(t)$ described in equation (7). Budgets can be handled as random variables with some given probability distribution;

- The credibility factors of experts (or scenarios) can also be handled as random variables within some interval. For example the credibility of the $i$-th expert can be assessed as being a normal distribution with some mean value and standard deviation (it must be truncated to avoid negative values or values larger than one).

Simulation codes can generate results in the form of probability distributions of key variables in the model, evidencing percentiles.

We think, however, that such refinements will have limited added values in terms of insight gained by the authorities. In addition a range of possible strategies are obtained, and not a single one. This raises new questions on which one has to be eventually adopted. In our opinion it is more adequate to have a final decision over a limited time-horizon, say five years. In this way the strategy can be periodically revisited if necessary to better match the objectives.

At each time step, a fuzzy-reasoning process takes place in the way shown in figure 7. This first model was static. For our purpose it can easily be made dynamic by combining it with the SD model presented in figure 3: in each time value, the computed values of the variables from the previous time step are used for forward computing in the following time step. We used for this time simulation the system dynamics code VENSIM® DSS32 (2000) for reasons of convenience. This code is easy to use, it has an easy-to-understand graphical interface, and it permits to work with vectors (subscripts), which is useful for large $n$, and $L$ values. VENSIM® also has the capability of performing sensitivity analyses using random drawings from given probability distributions of model parameters. The equations of this model are available from the authors on request.

(Other simulation tools could be used, e.g. SIMULINK®, which can be easily combined with the mentioned Fuzzy Toolbox of MATLAB® (op. cit.), but such development may require more programming skills).

We do not use any real data for this simulation. For a more realistic model, the assumptions on the number of experts and technologies can be changed as wished. Also the assumption on constant credibility factors is easy to relax.

Figure 8 shows the additional VENSIM view, which is added to the model to include the fuzzy reasoning with two experts we have presented here.

Figure 9 shows the final results with two experts ($C_1=0.5; C_2=0.7$) for comparison with figure 5. By comparing both figures it can be immediately seen that a similar reduction of fossil fuel can be achieved for a much smaller increase of the price, and thus a far smaller impact on the interest rate, when the opinions of both experts are taken into account.

6. Conclusions

In order to design a coherent and efficient strategic plan for a CO$_2$ tax scheme over a medium-term horizon, e.g. five years, decision-aiding instruments must be developed.

It is the purpose of our approach to assist and formalise the preparation of this strategic plan, given that are many different opinions about key variables or relationships like the exact shape of marginal-cost curves or the functional relationship between fossil-fuel price and interest rates.
Figure 8. The additional view in VENSIM to calculate the aggregated impact function in presence of two experts.

Figure 9. The results of the simulation for two experts ($C_1=0.5; C_2=0.7$) (see the legends of figure 5, the corresponding figure for one expert $C_1=0.5$).

What can be used here is scenario analysis and a rolling plane horizon of five years, i.e., with updates to be made yearly in the projections and paths for the five coming years. Classical scenario or sensitivity analysis is not entirely satisfactory, however. It provides ranges of values, elaborating on optimistic, average, or pessimistic forecasts related to the evolution of the output variables in the model. But collapsing many scenarios into one requires an additional knowledge of a priori occurrence probabilities, which is almost never there.

Another difficulty is that most CO$_2$-control models are static and deterministic, and thus not suitable for strategic planning. In the present paper we have presented a dynamic model aggregating an arbitrary number of opinions regarding key parameters in the model: MC cost curves of fossil-fuel reduction, and macroeconomic impact of the fuel price with tax on the interest rate. A didactic example relative to the residential sector has been presented in details.
The central driver of this model is based on fuzzy-reasoning rather than on probabilistic risk approaches to account for uncertainties on key parameters and functional relationships in the model. We feel that in technically complex frameworks, like CO₂-emission control in the residential sector, it proves to be extremely difficult, if not impossible, to use time series from the past to extrapolate development in a rather distant future. Many past data, especially in the field of technology development or macroeconomic behaviours, are quite useless for forecasts. It is why the authors think that it is important to use the concept of ‘possibility’, central to fuzzy reasoning, rather than the concept of ‘probability’, central to the Bayesian approach of subjective probabilities. Imprecise statements like ‘small’, ‘large’, etc. are indeed here better captured by membership functions than by probability values.

Note that fuzzy logic can serve whenever there is ambiguity and imprecision with respect to numerical data to be used in any simulation model. In Kunsch and Fortemps (2002), one of the authors has shown an example of a two-stage fuzzy inference system (FIS). The ‘maturity level’ of a technology is used as input to the first FIS: it provides as the final output of rules the membership function of the possible technology cost range. Because the maturity level is itself a fuzzy concept, a second FIS is developed where it appears in turn as the output. The future R&D budget needed for the further development of the technology is used as input to this second FIS. As in the present paper expert opinions with different credibility factors are used to assess the R&D budget.

In the present model, we think that additional development is possible, upstream of the FIS we have presented. The idea is to better evaluate important input variables or parameters which we assumed so far to be given, like:

- The credibility of expert, which should indeed result from a previous analysis, one basic input to the FIS which needs further validation;

- The yearly budgets, which although they were not considered in the simple didactic model we have presented, may impose an important constraint on residential investors.

As a last remark, we think that a model used for planning purposes in uncertain futures, and not amenable to probabilistic treatment, must rely on a regular updating of data (see for example Brans et al. (2001)). It is why it is recommended to consider a rather short rolling horizon, five years being a reasonable assumption.

References


